KÄNGURU DER MATHEMATIK 2019
21. 3. 2019

Level: Junior, Grade: 9 - 10

Name:	
School:	
Class:	

Time: 75 min.
30 starting points
each correct answer to questions 1. – 10.: 3 points
each correct answer to questions 11. – 20.: 4 points
each correct answer to questions 21. – 30.: 5 points
each question left unanswered: 0 points
each incorrect answer: minus ¼ of the points for the question

Please write the letter (A, B, C, D, E) of the correct answer in the square under the question number (1 bis 30). Write clearly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

| 21	22	23	24	25	26	27	28	29	30

- 3 Point Examples -

1. $20 \times 19 + 20 + 19 =$

 (A) 389   (B) 399   (C) 409   (D) 419   (E) 429

2. A model railway goes round in circles. It drives with constant speed and needs exactly 1 minute and 11 seconds for one circuit. How long does it need for six circuits?

(A) 6 minutes 56 seconds   (B) 7 minutes 6 seconds   (C) 7 minutes 16 seconds
(D) 7 minutes 26 seconds   (E) 7 minutes 36 seconds

3. A barber wants to write the word SHAVE on a board so that a customer who sees the word in the mirror can read the word normally. How does he have to write the word on the board?

4. How many different sums of the dots can one obtain if three ordinary dice are thrown at the same time?

 (A) 14   (B) 15   (C) 16   (D) 17   (E) 18

5. Five identical measuring jugs are filled with water. Four of them contain exactly the same amount of water. Which measuring jug contains a different amount?

(A)   (B)   (C)   (D)   (E)
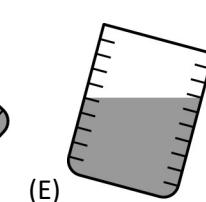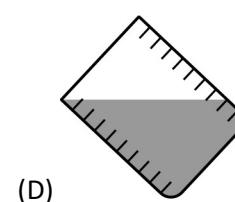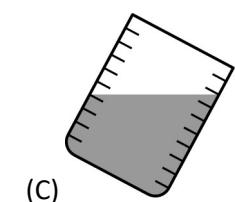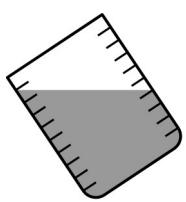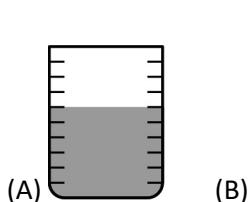
6. A park has five entrances. Monika wants to enter the park through one entrance and leave the park through another entrance. How many ways are there in which she can enter and leave the park?

 (A) 25   (B) 20   (C) 16   (D) 15   (E) 10

7. The individual masses (in kg) of three kangaroos are three different integers. Together they weigh 97 kg. What is the maximum weight the lightest of the three can have?

 (A) 1   (B) 30   (C) 31   (D) 32   (E) 33

8. Which of the following statements is definitely true for the angle marked in the diagram which is made up of nine squares?

(A) $\alpha = \beta$ (B) $2\,\alpha + \beta = 90°$ (C) $\alpha + \beta = 60°$ (D) $2\,\beta + \alpha = 90°$ (E) $\alpha + \beta = 45°$

9. Inside a unit square a certain area has been coloured in black. In which square is the black area biggest?

(A)   (B)   (C)   (D)   (E)
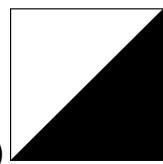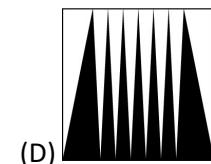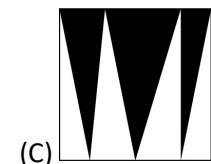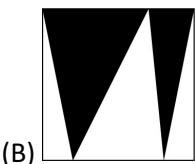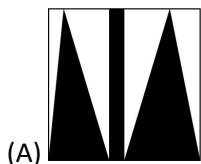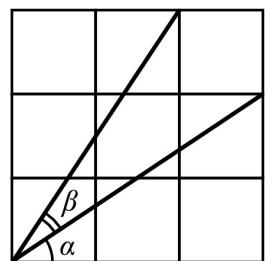
10. Julia reads a book whose pages are all numbered. The digit 0 appears six times and the digit 8 seven times. What is the page number of the last page?

 (A) 58   (B) 68   (C) 70   (D) 78   (E) 98

11. Three five-digit numbers are written onto three separate pieces of paper as shown. Three of the digits in the picture are hidden. The sum of the three numbers is 57263. Which are the hidden digits?

 (A) 0, 2 and 2　　(B) 1, 2 and 9　　(C) 2, 4 and 9　　(D) 2, 7 and 8　　(E) 5, 7 and 8

12. Anna, Bella, Claire, Dora, Erika and Frieda meet at a party. Each pair who know each other shake hands exactly once. Anna shakes hands only once, Bella twice, Claire three times, Dora four times and Erika five times. How many people does Frieda shake hands with?

 (A) 1　　　　　(B) 2　　　　　(C) 3　　　　　(D) 4　　　　　(E) 5

13. The vertices of a square *ABCD* are labelled anti-clockwise. *A* and *C* are the vertices of an equilateral triangle *AEC*, whose vertices are also labelled anti-clockwise.
How big is the angle *CBE*?

 (A) 30°　　　　(B) 45°　　　　(C) 135°　　　　(D) 145°　　　　(E) 150°

14. The numbers a, b, c and d are pairwise different integers between 1 and 10 (1 and 10 including).
What is the smallest possible value of the expression $\frac{a}{b} + \frac{c}{d}$?

 (A) $\frac{2}{10}$　　　　(B) $\frac{3}{19}$　　　　(C) $\frac{14}{45}$　　　　(D) $\frac{29}{90}$　　　　(E) $\frac{25}{72}$

15. The flag of Kanguria is a rectangle whose side lengths are in the ratio $3:5$.
The flag is split into four rectangles of equal area as shown.
In which ratio are the side lengths of the white rectangle?

 (A) $1:3$　　　　(B) $1:4$　　　　(C) $2:7$　　　　(D) $3:10$　　　　(E) $4:15$

16. A 3×2 rectangle can be covered in two ways by two of the L-shaped figures as shown:
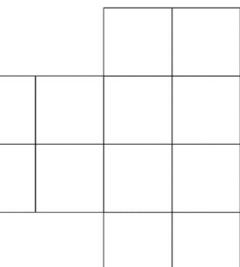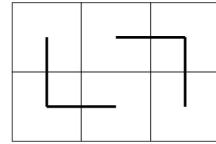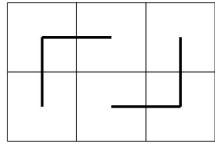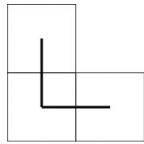
 In how many ways can the diagram on the right be covered by these L-shaped figures?

 (A) 1　　　　　(B) 2　　　　　(C) 3　　　　　(D) 4　　　　　(E) 48

17. A triathlon consists of three disciplines swimming, running and cycling. The cycle route is three quarters of the entire distance, the running route is one fifth of the entire distance and the swimming route is 2 km long. How long is the whole distance of the triathlon in km?

 (A) 10　　　　(B) 20　　　　(C) 38　　　　(D) 40　　　　(E) 60

18. A 1-liter-bottle of syrup is still half full. The syrup shall be diluted in the ratio $1:7$ to make juice.
Which fraction of the syrup should be used to obtain 2 litres of juice?

 (A) $\frac{1}{4}$　　　　(B) $\frac{1}{2}$　　　　(C) $\frac{2}{7}$　　　　(D) $\frac{4}{7}$　　　　(E) the whole syrup

19. The diagram consists of three circles of equal radius R. The centre of those circles lie on a common straight line where the middle circle goes through the centres of the other two circles (see diagram). How big is the perimeter of the figure?
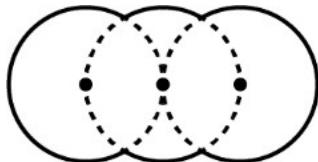
 (A) $\frac{10\pi R}{3}$　　　　(B) $\frac{5\pi R}{3}$　　　　(C) $\frac{2\pi R\sqrt{3}}{3}$　　　　(D) $2\pi R\sqrt{3}$　　　　(E) $4\pi R$

20. The sum of the seven digits of a seven-digit phone number $aaabbbb$ is a two-digit number ab.
How big is the value of the sum $a + b$?

 (A) 8　　　　(B) 9　　　　(C) 10　　　　(D) 11　　　　(E) 12
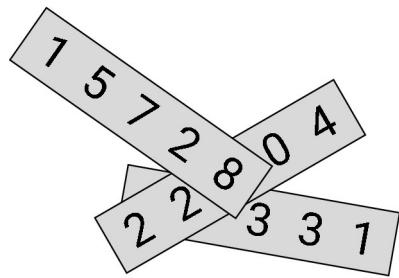
21. If one of the digits of a two-digit number is deleted, the result in both cases is a factor of the original number. How many two-digit numbers have this property?

(A) 5   (B) 9   (C) 14   (D) 19   (E) 23

22. 60 apples and 60 pears in total are shared out in several boxes. There should be the same amount of apples in each box but no two boxes should contain the same amount of pears. Each box contains both fruits. What is the maximum number of boxes that can be filled in this way?

(A) 20   (B) 15   (C) 12   (D) 10   (E) 6

23. The diagram shows the net of an octahedron. Which edge meets the edge labelled with x if the net is folded up to form an octahedron?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

24. Two vertices of a square lie on a semi-circle as shown, while the other two lie on its diameter. The radius of the circle is 1 cm. How big is the area of the square?

(A) $\frac{4}{5}$ cm²   (B) $\frac{\pi}{4}$ cm²   (C) 1 cm²   (D) $\frac{4}{3}$ cm²   (E) $\frac{2}{\sqrt{3}}$ cm²

25. Two points are marked on a circular disc that rotates about its centre. The outer point is 3 cm further away from the centre than the inner point and it moves 2.5 times as fast as the inner point. How big is the distance between the outer point and the centre of the circular disc?

(A) 10 cm   (B) 9 cm   (C) 8 cm   (D) 6 cm   (E) 5 cm

26. The integers from 1 to 99 are written down in ascending order without a gap. This sequence of numbers is divided up into triples (groups of three):
$123456789101112\cdots979899 \rightarrow (123)(456)(789)(101)(112)\cdots(979)(899)$.
Which of the following triples is not obtained?

(A) (222) (B) (444) (C) (464) (D) (646) (E) (888)

27. How many planes exist that go through exactly three vertices of a given cube?

(A) 2   (B) 4   (C) 6   (D) 8   (E) 10

28. A graph consists of 16 points and several connecting lines as shown in the diagram. An ant is at point A. With every move the ant can move from the point where it currently is, along one of the connecting lines, to an adjacent point.
At which of the points P, Q, R, S and T can the ant be after 2019 moves?

(A) only at P, R or S, not at Q or T   (B) only at P, R, S or T, not at Q
(C) only at Q   (D) only at T   (E) At all of the points

29. The numbers a, b and c are three-digit numbers and in each number the first digit is equal to the last one. Furthermore $b = 2a + 1$ and $c = 2b + 1$. How many possible values are there for the number a?

(A) 0   (B) 1   (C) 2   (D) 3   (E) more than 3

30. What is the minimum number of elements of the set $\{10, 20, 30, 40, 50, 60, 70, 80, 90\}$ that have to be removed so that the product of the remaining elements of the set is a square number?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5